A comparison of probabilistic population code and sampling-based code in neural estimations of time-varying quantities

Siqi Zhou (siqi.zhou@mail.utoronto.ca)
University of Toronto Institute for Aerospace Studies, 4925 Dufferin Street
Toronto, ON M3H 5T6 Canada

Abstract

When driving or playing sports, despite the presence of non-deterministic factors, the brain is required to reliably estimate the positions or velocities of objects to plan for subsequent actions. In the literature, with Bayesian as the mathematical framework and probabilistic population code (PPC) as the neural representation model, neural circuits for computations such as multi-sensory cue integration and odour identification have been discussed; however, less attention has been given to comparisons with alternative neural representations, such as the sampling-based code, especially, for inference problems that are time-variant in nature. In this work, with the motivation of exploring neural probabilistic inferences and specific focus on inferences of time-varying quantity estimations, plausible neural circuits derived based on the PPC and sampling-based code are examined. Based on numerical comparisons, it is found that, with less constraints on the form of probabilistic functions being represented, the sampling-based code is an efficient alternative to the PPC for modelling neural approximate Bayesian inferences in estimation problems.

Keywords: Neural representations; Bayesian approximation; computational neuroscience

Introduction

There has been a range of studies on probability representations and inferences utilized by the brain for optimizing decisions. As noted in the review of (Pouget, Beck, Ma, & Latham, 2013), among different neural representation proposals, two of them, originated with very different assumptions, are the probabilistic population code (PPC) and sampling-based code. The former assumes the neural activities of a population of neurons collectively encode parameters of probability distributions, while the latter assumes neural activities represent sampled values drawn from underlying probability distributions (Pouget et al., 2013). The fundamental differences in these proposals set an interesting background for designing numerical comparisons to explore their inherent properties in higher-level inference tasks.

With the focus of examining marginalization computations implemented by the brain, based on the linear PPC proposal, neural dynamics that approximate the Kalman Filter (KF) for position estimation problems are derived in (Beck, Latham, & Pouget, 2011). Motivated by the discussions in the paper, this work further explores neural computation models for time-varying quantity estimation problems.

In the literature, the sampling-based proposal has been studied and compared with the PPC proposal for general perception and olfaction inference problems; however, less attention has given to quantity estimations – inferring quantities based on noisy sensory information. In (Fiser, Berkes, Orbán, & Lengyel, 2010), for general perception problems, the feasibility of implementing short-term inference and long-term learning based on the PPC and sampling-based proposals are discussed. It is concluded that the sampling-based code is more efficient for encoding arbitrary probability distributions and is more suited for learning. In addition, in (Grabska-Barwinska, Beck, Pouget, & Latham, 2013), for olfactory inferences, it is found that these two neural representation models lead to similar demixing performance but different odour concentration predictions. Along this line of exploration, with the formulation in (Beck et al., 2011) as the starting point, this work provides insights into low-level neural computational models specific to time-varying quantity estimations, and explores the differences of the PPC and sampling-based proposals in this class of inference problem.

Computational framework and models

Bayesian inference is a framework that can characterize the complex computations carried out in the brain (Pouget et al., 2013). For estimation problems considered in this work, the general model of Bayesian inference (i.e., the Bayes Filter) is of the following form

$$\underbrace{p(\mathbf{s}(t)|\mathbf{r}_{\text{in}}(0),...,\mathbf{r}_{\text{in}}(t))}_{\substack{\text{posterior encoded by}\\\text{output layer}}} = \eta\, \underbrace{p(\mathbf{r}_{\text{in}}(t)|\mathbf{s}(t))}_{\substack{\text{generative model}\\\text{encoded by}\\\text{input layer}}} \int \underbrace{p(\mathbf{s}(t)|\mathbf{s}(t-\Delta t))}_{\substack{\text{internal dynamic}\\\text{model available}\\\text{to the brain}}} \times p(\mathbf{s}(t-\Delta t)|\mathbf{r}_{\text{in}}(0),...,\mathbf{r}_{\text{in}}(t-\Delta t))\, d\mathbf{s}(t-\Delta t),$$

where $\mathbf{s}$ is the vector of quantities (or, stimuli) to be inferred, $\mathbf{r}$ is the vector of activities of a neuron population, and η is the normalization constant (Beck et al., 2011). For neural implementations, the propagation component $p(\mathbf{s}(t)|\mathbf{s}(t-\Delta t))$ is realized with the assumption that the brain itself has an internal dynamic model that provides a prior estimation of the stimuli (Wolpert, Ghahramani, & Jordan, 1995), the generative component $p(\mathbf{r}(t)|\mathbf{s}(t))$ is encoded by a population of neurons (denoted by subscript 'in') to correct the prior predictions, and the posterior $p(\mathbf{s}(t)|\mathbf{r}(0),...,\mathbf{r}(t))$ is encoded by a second population of neurons (denoted by subscript 'out') and is propagated based the predictive and corrective steps described above. With this structure as the basis, neural approximate Bayes Filters based on the PPC and sampling-based code are compared below.

PPC-based model

In (Beck et al., 2011), with the linear PPC, firing rate dynamics for the following estimation problem is derived:

$$\frac{ds(t)}{dt} = -\alpha s(t) + w(t), \tag{1}$$

where $s(t)$ is the position to be estimated and $w(t) \sim \mathcal{N}(0, \sigma_w^2)$ is Gaussian motion noise, and α is a constant. In this formulation, with Poisson activities and Gaussian tuning curves, the populations of neurons encode observations and posterior beliefs about $s(t)$ as Gaussian distributions $\mathcal{N}(\mu(t), \sigma^2(t))$. Given these assumptions, the neural activities in the output layer can be related to that in the input layer as follows (Beck et al., 2011):

$$\frac{d\mathbf{v}_{\text{out}}(t)}{dt} = \underbrace{\alpha \mathbf{W} \rho_{\text{out}}(t) - \sigma_w^2 (\mathbf{a}_{\text{out}}^T \rho_{\text{out}}(t)) \mathbf{v}_{\text{out}}(t)}_{\text{predictive step}} + \underbrace{\mathbf{M} \rho_{\text{in}}(t)}_{\text{corrective step}} \ ,$$

where $\mathbf{v}_{\text{out}}$ is the firing rates of the neurons in the output layer, ρ_{in} and ρ_{out} are the spike trains generated by the neurons in the input and output layers, $\mathbf{W} = 2\mathbf{a}_{\text{out}}^{\dagger} \mathbf{a}_{\text{out}}^T + \mathbf{b}_{\text{out}}^{\dagger} \mathbf{b}_{\text{out}}^T$ and $\mathbf{M} = \mathbf{a}_{\text{out}}^{\dagger} \mathbf{a}_{\text{in}}^T + \mathbf{b}_{\text{out}}^{\dagger} \mathbf{b}_{\text{in}}^T$, and the $\mathbf{a}$ and $\mathbf{b}$ are parametrization vectors such that $\mathbf{a} \cdot \mathbf{r}(t) = \frac{1}{\sigma^2(t)}$ and $\mathbf{b} \cdot \mathbf{r}(t) = \frac{\mu(t)}{\sigma^2(t)}$.

Sampling-based model

Motivated by the Particle Filter algorithm in robotics applications, an alternative neural estimator with the sampling-based code is derived. In this formulation, the activities of neurons represent number of samples at their preferred stimuli. For comparison purposes, by assuming $\mathbf{v}_{\text{out}} = f(\mathbf{r}_{in}(t), \mathbf{r}_{\text{out}}(t))$ as in the PPC-based formulation, the output layer neural dynamics can be modelled by

$$\frac{d\mathbf{v}_{\text{out}}(t)}{dt} = \underbrace{\mathbf{P} \rho_{\text{out}}(t)}_{\text{predictive step}} + \underbrace{\mathbf{Q} \rho_{\text{in}}(t)}_{\text{corrective step}} \ ,$$

where with s^0 denoting preferred stimuli and l representing a sensitivity measure from activities in the input layer to the output layer, the matrices $\mathbf{P}_{ij} = \exp\left(-\frac{[s^0_{\text{post,i}} - (1-\alpha\Delta t) s^0_{\text{post,j}}]^2}{2\sigma_w^2}\right) - \delta(s^0_{\text{post,i}} - s^0_{\text{post,j}})$ and $\mathbf{Q}_{ij} = \exp\left(-\frac{(s^0_{\text{post,i}} - s^0_{\text{in,j}})^2}{2l^2}\right) - \left[1 - \exp\left(-\frac{(s^0_{\text{post,i}} - s^0_{\text{in,j}})^2}{2l^2}\right)\right]$ are chosen to approximate the prediction and correction steps in the Particle Filter algorithm (Barfoot, 2017).

Results and conclusions

For the numerical example in (1), the neural computational models derived based on the PPC and sampling-based code are compared against the standard Kalman Filter algorithm, which provides the optimal solution to the Bayes Filter for this linear estimation problem. As can be seen from Figure 1, in comparison, the sampling-based code leads to estimations closer to the optimal solution. With a closer look at the formulations, this difference can be traced back to the derived weights. For the PPC-based formulation, depending on the choices of the parametrization vectors $\mathbf{a}$ and $\mathbf{b}$, the output neurons may be more sensitive to input neurons with very different preferred stimuli. This results in inefficient flow of information from the input to the output layer and hence less optimal estimations. In contrast, for the sampling-based code, with more flexibilities in representing probability functions, this insufficiency is alleviated.

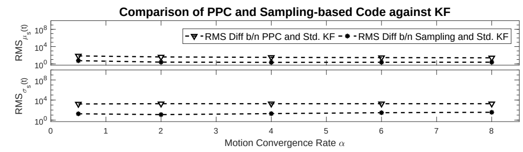

Figure 1: Root mean square (RMS) difference in estimation mean and variance between the neural estimators and the standard Kalman Filter for $\alpha = \{0.5, 2.0, 4.0, 6.0, 8.0\}$.

In order to further explore the robustness of these neural estimation models, a separate set of numerical comparisons is carried out to simulate the scenario where observations (encoded by the input layer) are temporarily disrupted (e.g., momentarily blocked vision when driving). It is found that, for both computational models, the recovery time from the disruption is on the order of milliseconds. When comparing the estimations after the disruption, the sampling-based model has a slight increase in error and some decrease in confidence, which are consistent with day-to-day behaviour level observations.

The neural circuits discussed in this work are for the 1D position estimation problem. For more general inferences, additional layers of neurons can be included in the structure to account for higher dimensions (Pouget et al., 2013) or decomposing the output layer into novelty and filtering layers to allow for additional incorporations of control inputs (Kutschireiter, Jean, & Pfister, 2015).

References

Barfoot, T. D. (2017). *State estimation for robotics - a matrix lie group approach*. Cambridge University Press. doi: 10.1017/9781316671528

Beck, J. M., Latham, P. E., & Pouget, A. (2011). Marginalization in neural circuits with divisive normalization. *Journal of Neuroscience*, *31*(43), 15310–15319.

Fiser, J., Berkes, P., Orbán, G., & Lengyel, M. (2010). Statistically optimal perception and learning: from behavior to neural representations. *Trends in cognitive sciences*, *14*(3), 119–130.

Grabska-Barwinska, A., Beck, J., Pouget, A., & Latham, P. (2013). Demixing odors-fast inference in olfaction. In *Advances in neural information processing systems* (pp. 1968–1976).

Kutschireiter, A., Jean, S. C. S. H. S., & Pfister, P. (2015). The neural particle filter. *arXiv preprint arXiv:1508.06818*.

Pouget, A., Beck, J. M., Ma, W. J., & Latham, P. E. (2013). Probabilistic brains: knowns and unknowns. *Nature neuroscience*, *16*(9), 1170–1178.

Wolpert, D. M., Ghahramani, Z., & Jordan, M. I. (1995). An internal model for sensorimotor integration. *Science*, *269*(5232), 1880.