A Comparison of Probabilistic Population Code and Sampling-Based Code in Neural Estimations of Time-Varying Quantities

SiQi Zhou | University of Toronto

Motivation

- In daily activities (e.g., driving and playing sports), despite the presence of uncertainties, the brain needs to reliably estimate time-varying quantities such as the position and velocity of objects.
- *Bayesian inference* is used to explain the near-optimal decision-makings achieved by the brain.
- The *probabilistic population code* (PPC) and *sampling-based code* (SBC) are neural probability representation proposals originated from very different assumptions [1].
- These proposals have been compared for problems such as de-mixing odors [2]; less attention has given to problems that are time-variant in nature.

Bayes Filter

- Bayesian inference model for time-varying quantity estimation (the Bayes Filter) [3]

$$\underbrace{p(\mathbf{s}(t)|\mathbf{r}_{\text{in}}(0),...,\mathbf{r}_{\text{in}}(t))}_{\substack{\text{posterior encoded by}\\\text{output layer}}} = \eta\, \underbrace{p(\mathbf{r}_{\text{in}}(t)|\mathbf{s}(t))}_{\substack{\text{generative model}\\\text{encoded by}\\\text{input layer}}} \int \underbrace{p(\mathbf{s}(t)|\mathbf{s}(t-\Delta t))}_{\substack{\text{internal dynamic}\\\text{model available}\\\text{to the brain}}} \times p(\mathbf{s}(t-\Delta t)|\mathbf{r}_{\text{in}}(0),...,\mathbf{r}_{\text{in}}(t-\Delta t))\, d\mathbf{s}(t-\Delta t),$$

Probabilistic Population Code (PPC)

- PPC: Neural activities of a population of neurons collectively encode parameters of probability distributions [4]

$$\mathbf{a}\cdot\mathbf{r}(t) = \frac{1}{\sigma^2(t)} \quad \text{and} \quad \mathbf{b}\cdot\mathbf{r}(t) = \frac{\mu(t)}{\sigma^2(t)}$$

- Benchmark problem: $\frac{ds(t)}{dt} = -\alpha s(t) + w(t)$
- Firing rate dynamics derived in [4]:

$$\frac{d\mathbf{v}_{\text{out}}(t)}{dt} = \underbrace{\alpha \mathbf{W} \boldsymbol{\rho}_{\text{out}}(t) - \sigma_w^2 (\mathbf{a}_{\text{out}}^T \boldsymbol{\rho}_{\text{out}}(t)) \mathbf{v}_{\text{out}}(t)}_{\text{predictive step}} + \underbrace{\mathbf{M}\boldsymbol{\rho}_{\text{in}}(t)}_{\text{corrective step}}$$

$$\mathbf{W} = 2\mathbf{a}_{\text{out}}^{\dagger}\mathbf{a}_{\text{out}}^T + \mathbf{b}_{\text{out}}^{\dagger}\mathbf{b}_{\text{out}}^T \quad \text{and} \quad \mathbf{M} = \mathbf{a}_{\text{out}}^{\dagger}\mathbf{a}_{\text{in}}^T + \mathbf{b}_{\text{out}}^{\dagger}\mathbf{b}_{\text{in}}^T$$

Optimal Estimation

- Kalman Filter (benchmark): Optimal estimation for linear motions with Gaussian noise [3]
- Prediction step:

$$\check{\sigma}_s^2(t) = \exp(-2\alpha\Delta t)\, \hat{\sigma}_s^2(t-\Delta t)$$
$$\check{\mu}_s(t) = \exp(-\alpha\Delta t)\, \hat{\mu}_s(t-\Delta t)$$

- Correction step:

$$K(t) = \frac{\check{\sigma}_s^2(t)}{\sigma_{\text{in}}^2(t) + \check{\sigma}_s^2(t)}$$
$$\hat{\sigma}_s^2(t) = (1-K(t))\, \check{\sigma}_s^2(t)$$
$$\hat{\mu}_s(t) = \check{\mu}_s(t) + K(t)\, (\mu_{\text{in}}(t) - \check{\mu}_s(t))$$

Sampling-Based Code (SBC)

- SBC: Neural activities represents sampled values from the underlying probability distribution [1].
- Motivation from Particle Filter (PF) algorithm: a sampling-based approach that provides solution to the Bayes Filter for nonlinear and non-Gaussian problems [3].
- Firing rate dynamics (approximating PF algorithm):

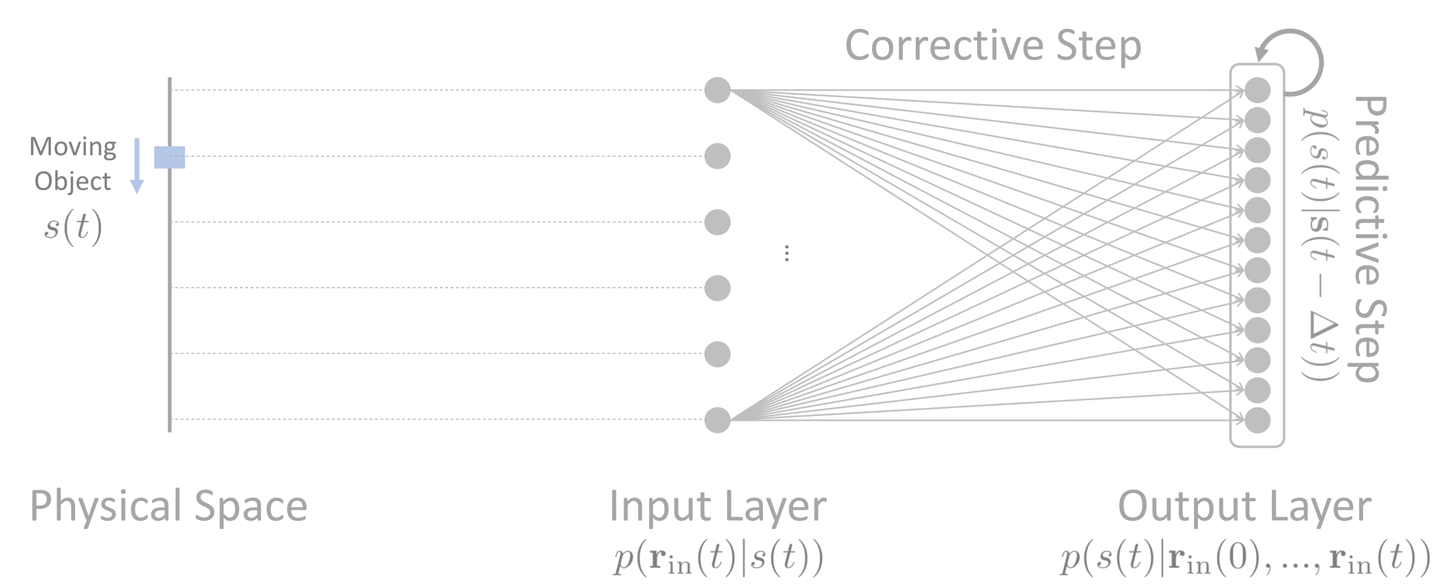
$$\frac{d\mathbf{v}_{\text{out}}(t)}{dt} = \underbrace{\mathbf{P}\,\boldsymbol{\rho}_{\text{out}}(t)}_{\text{predictive step}} + \underbrace{\mathbf{Q}\,\boldsymbol{\rho}_{\text{in}}(t)}_{\text{corrective step}}$$

$$\mathbf{P}_{ij} = \exp\left(-\frac{[s^0_{\text{post,i}} - (1-\alpha\Delta t)s^0_{\text{post,j}}]^2}{2\sigma_w^2}\right) - \delta(s^0_{\text{post,i}} - s^0_{\text{post,j}})$$

$$\mathbf{Q}_{ij} = \exp\left(-\frac{(s^0_{\text{post,i}} - s^0_{\text{in,j}})^2}{2l^2}\right) - \left[1 - \exp\left(-\frac{(s^0_{\text{post,i}} - s^0_{\text{in,j}})^2}{2l^2}\right)\right]$$

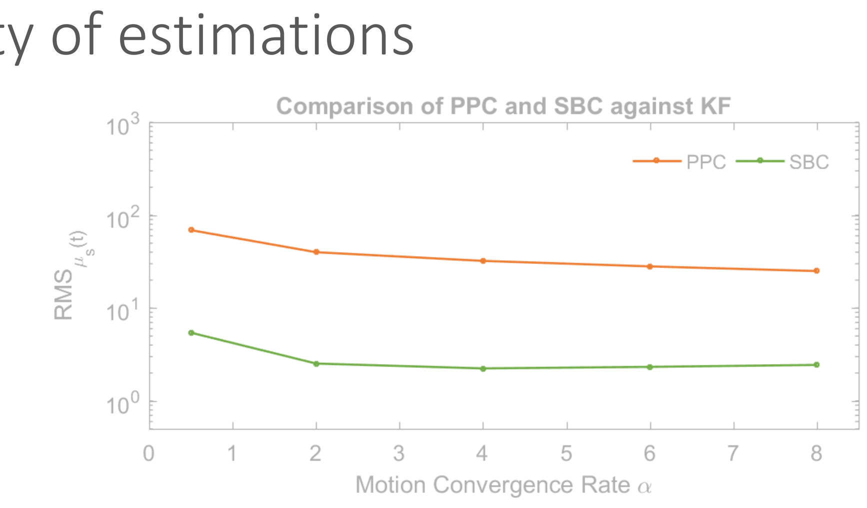
Neural Circuit

- Translation of the Bayes Filter to a neural circuit

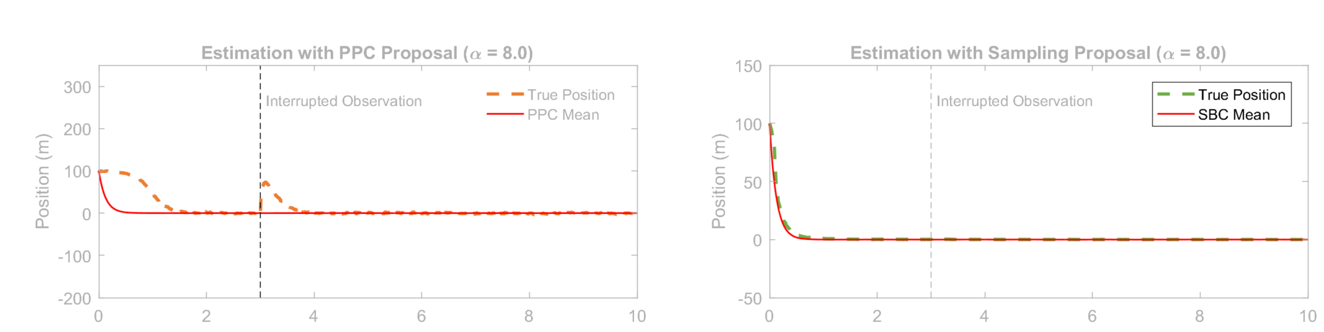


Results

- Optimality of estimations



- Recovery from interruptions



References

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[4] Beck, J. M., Latham, P. E., & Pouget, A. (2011). Marginalization in neural circuits with divisive normalization. Journal of Neuroscience, 31(43), 15310–15319.

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