

# Design of Deep Neural Networks as Add-on Blocks for Improving Impromptu Trajectory Tracking

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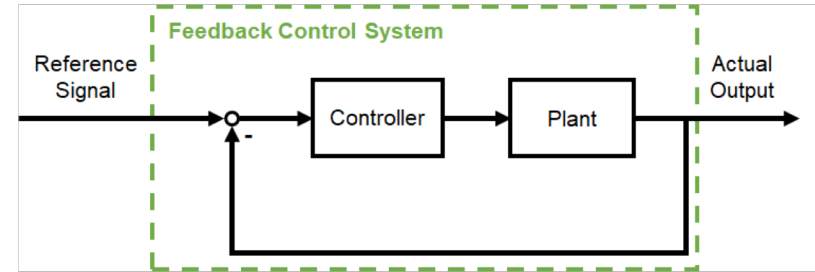
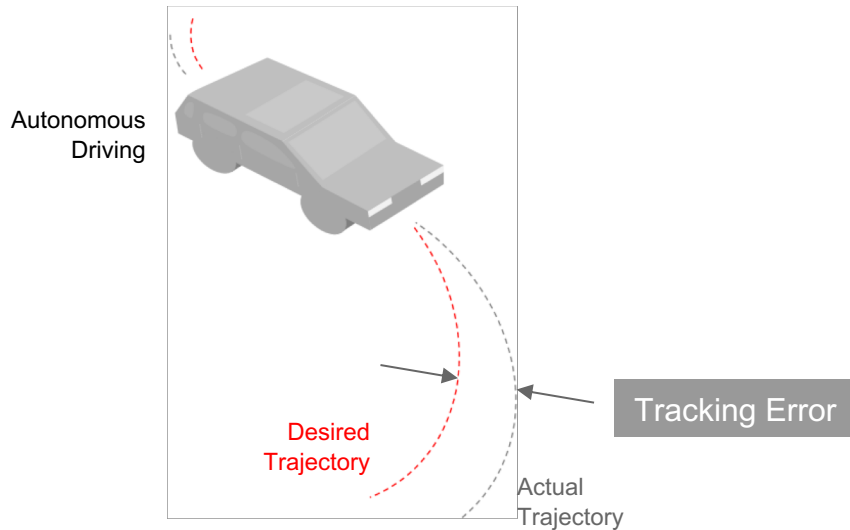
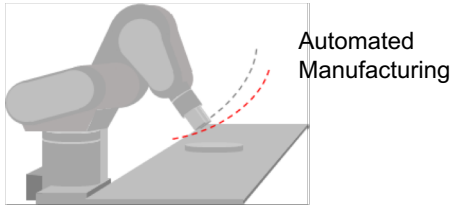
*Conference on Decision and Control (CDC) 2017*

SiQi Zhou, Mohamed K. Helwa, and Angela P. Schoellig

Dynamic Systems Lab | University of Toronto Institute for Aerospace Studies

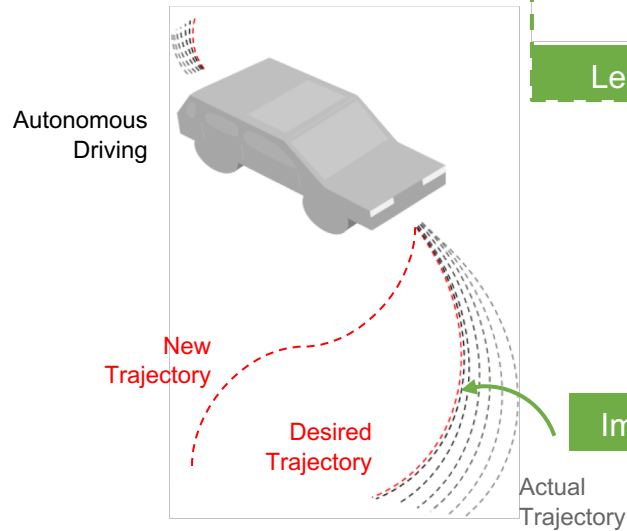
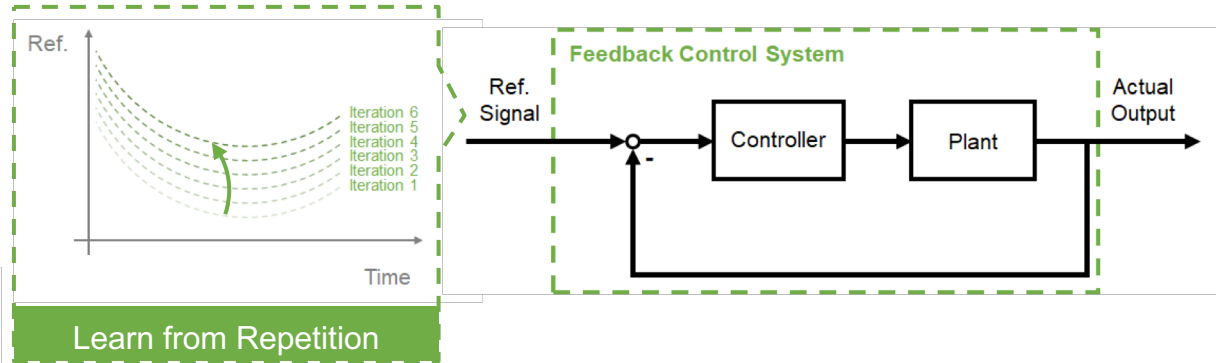
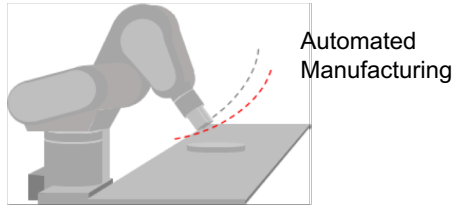


# Designing control systems for high-accuracy tracking is challenging

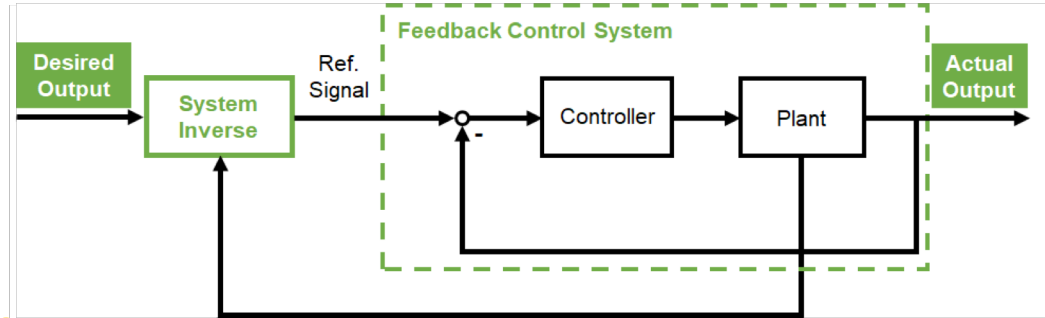
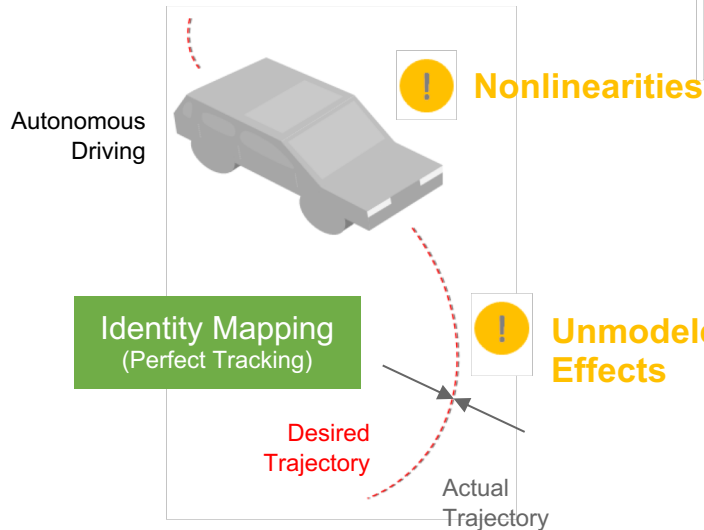
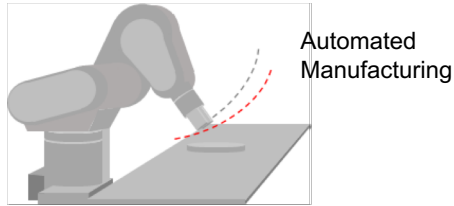


Perfect tracking cannot be achieved for arbitrary trajectories.

# Designing control systems for high-accuracy tracking is challenging



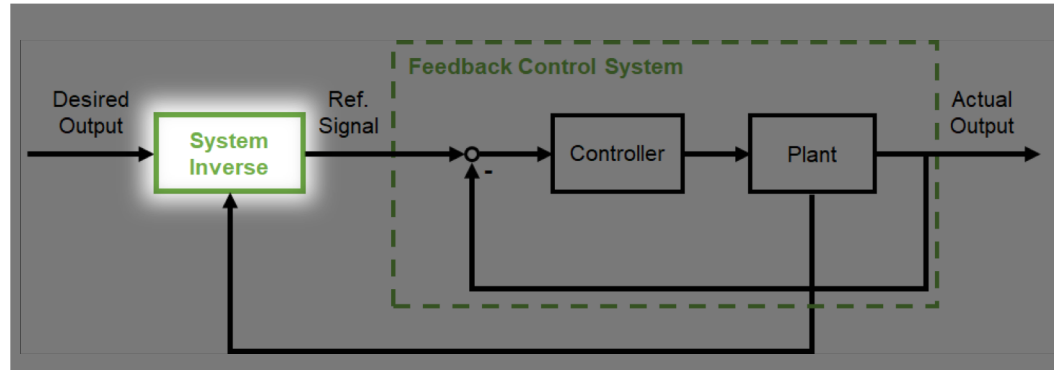
# Designing control systems for high-accuracy tracking is challenging



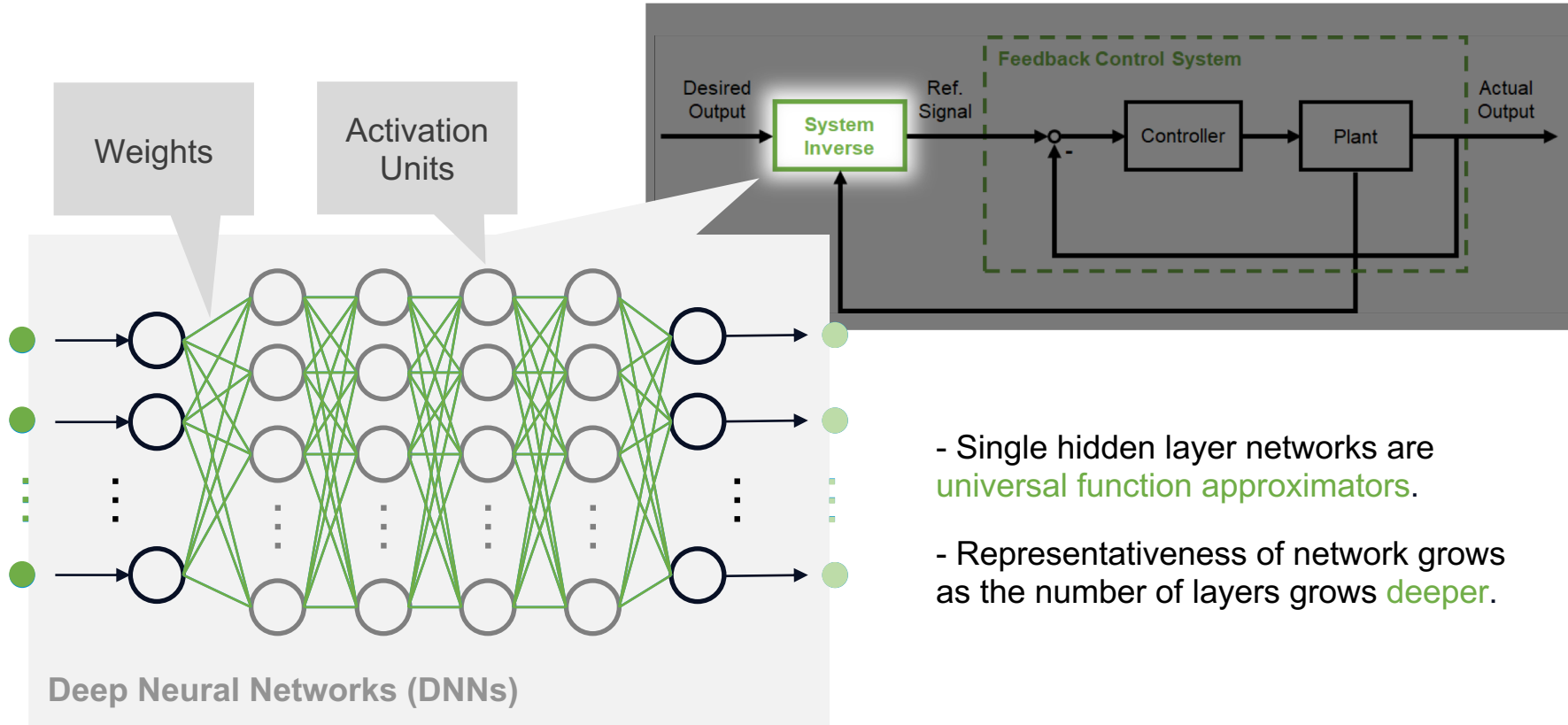
- Obtaining a sufficiently accurate inverse model is difficult in practice.
- Applying to non-minimum phase systems (i.e., systems with unstable inverse dynamics) is not trivial.

# Learning add-on blocks to enhance 'black-box' control systems

Learn **inverse** of closed-loop systems **from input-output data** to achieve high-accuracy impromptu tracking (i.e., tracking arbitrary trajectories in one shot)



# Deep Neural Networks (DNNs) as the learning technique

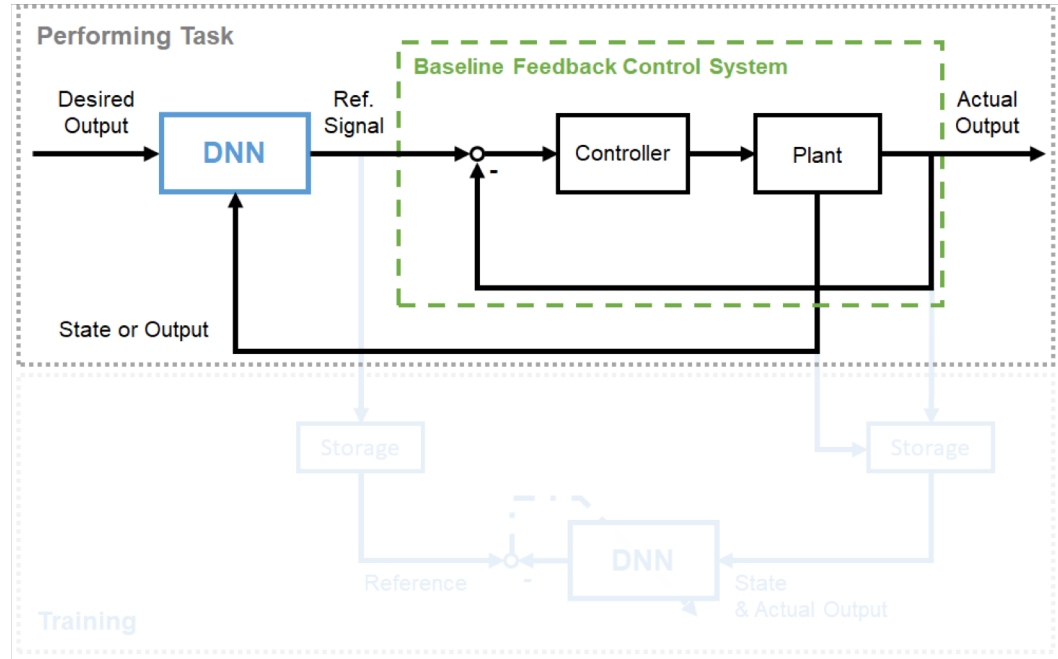


- Single hidden layer networks are **universal function approximators**.

- Representativeness of network grows as the number of layers grows **deeper**.

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Learn inverse of closed-loop systems from input-output data to achieve high-accuracy impromptu tracking (i.e., tracking arbitrary trajectories in one shot)

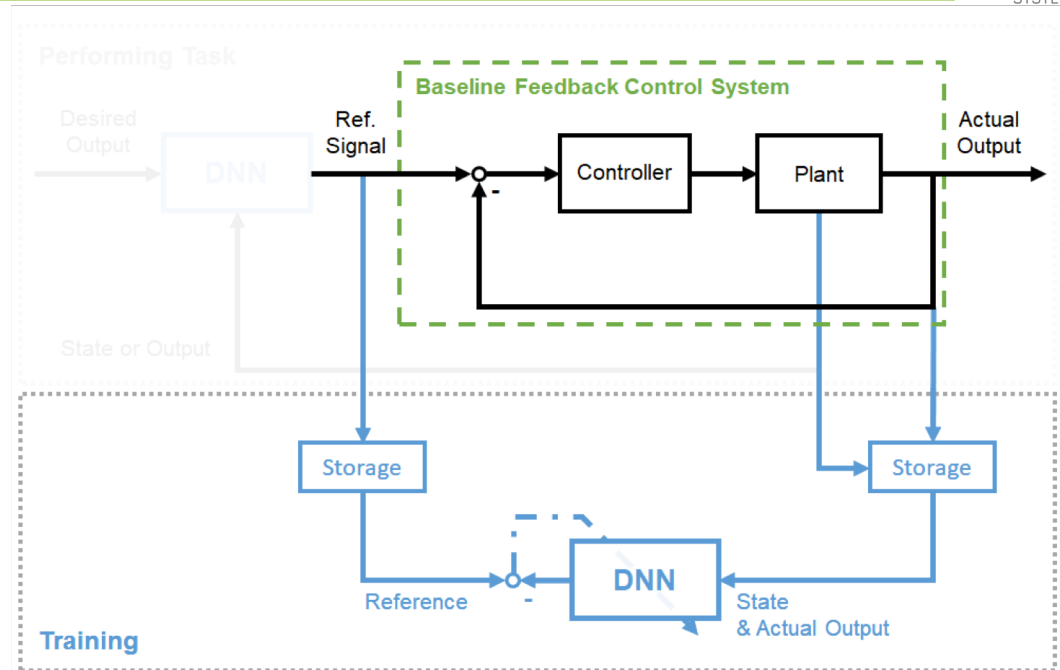


# DNN as add-on blocks to enhance ‘black-box’ control systems

Learn inverse of closed-loop systems from input-output data to achieve high-accuracy impromptu tracking (i.e., tracking arbitrary trajectories in one shot)

## Overview

- Training: a DNN module is trained with reversed input-output data of the baseline system.



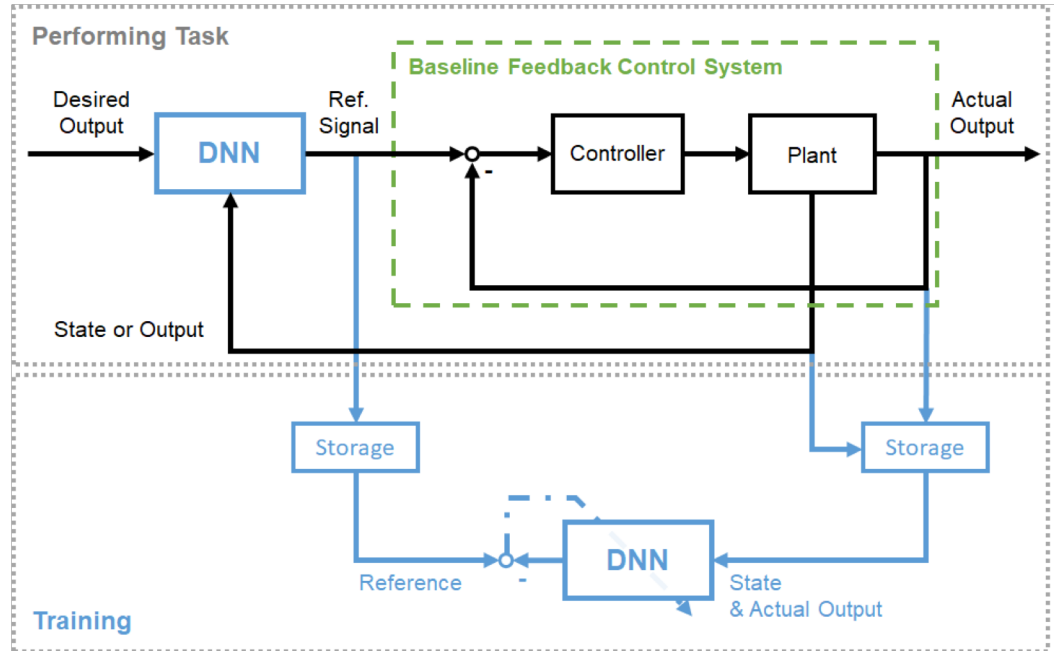


# DNN as add-on blocks to enhance ‘black-box’ control systems

Learn inverse of closed-loop systems from input-output data to achieve high-accuracy impromptu tracking (i.e., tracking arbitrary trajectories in one shot)

## Overview

- Training: a DNN module is trained with **reversed input-output data** of the baseline system.
- Performing task: the DNN add-on module **adjusts the reference** signal sent to the baseline system.



# The DNN add-on module reduces tracking error by 40%-50%

## Objective

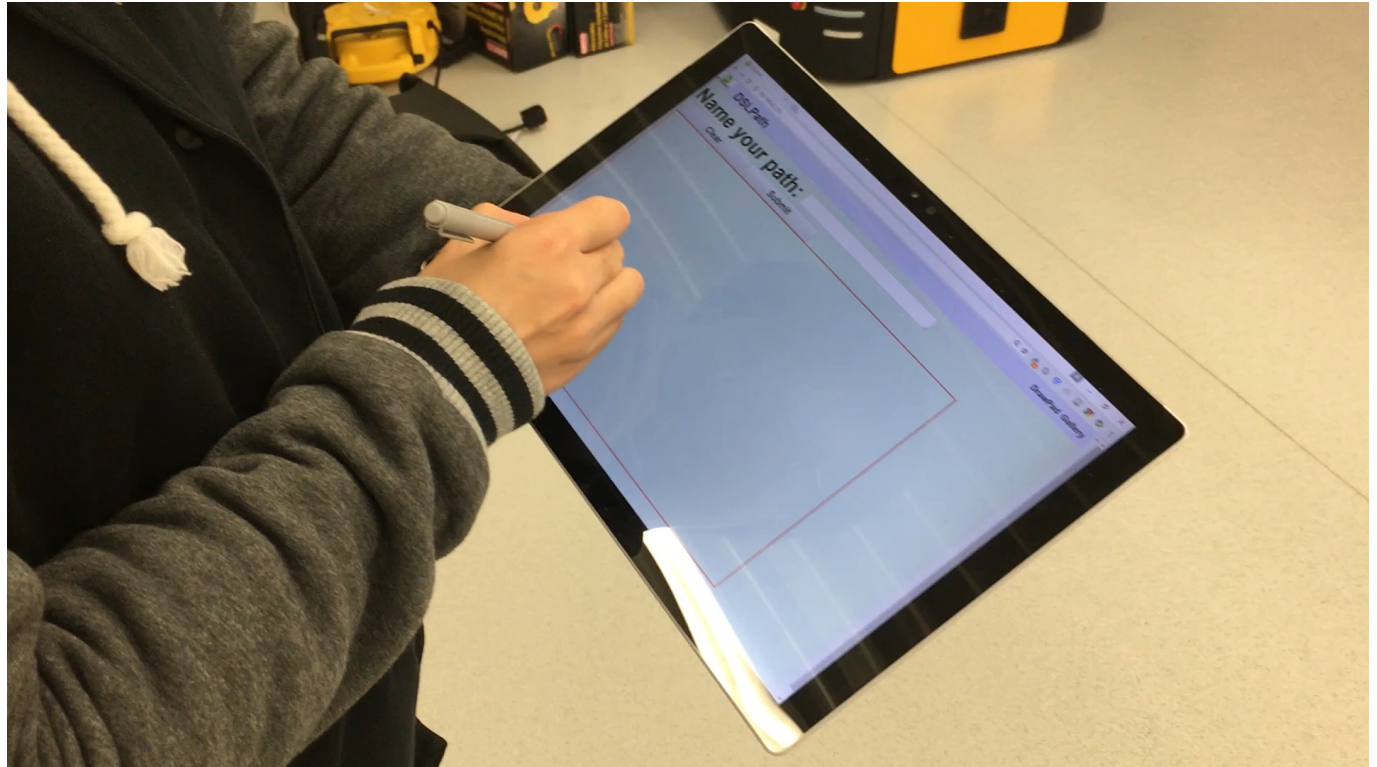
To track arbitrary hand-drawn trajectory with high-accuracy impromptu

## Procedure

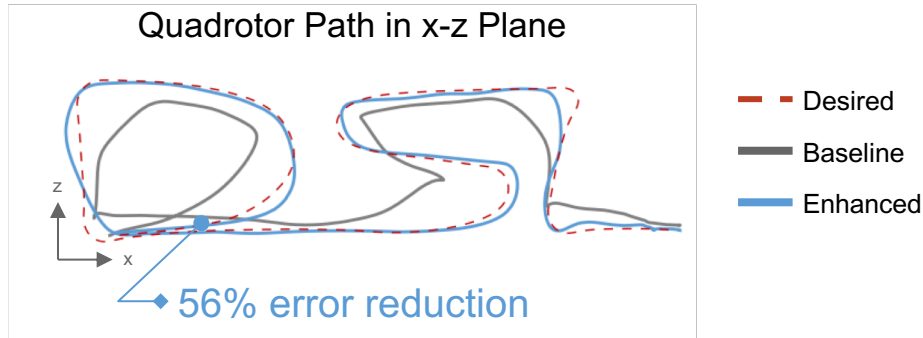
1. Collect data
2. Train network
3. Track hand-drawn trajectories

## Fly-as-You-Draw Project

Q. Li, J. Qian, Z. Zhu, X. Bao,  
M. K. Helwa, and A. P. Schoellig  
*"Deep Neural Networks for Improved,  
Impromptu Trajectory Tracking of  
Quadrotor"*  
(ICRA 2017)

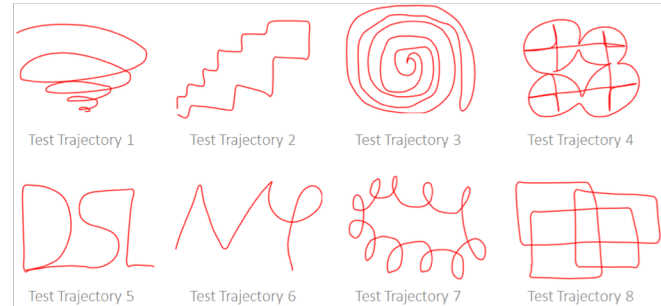


# The DNN add-on module reduces tracking error by 40%-50%

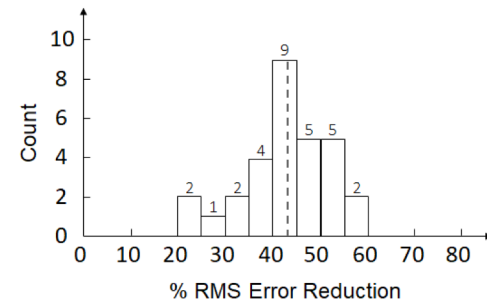


- 56% error reduction was achieved with only **20 min of training** on pure **sinusoidal trajectories**.
- On average of **30 hand-drawn trajectories**, **43% error reduction** was achieved.
- The dependent inputs of the DNN module were determined through **experimental trial-and-error**.

## Examples of Untrained Test Trajectories

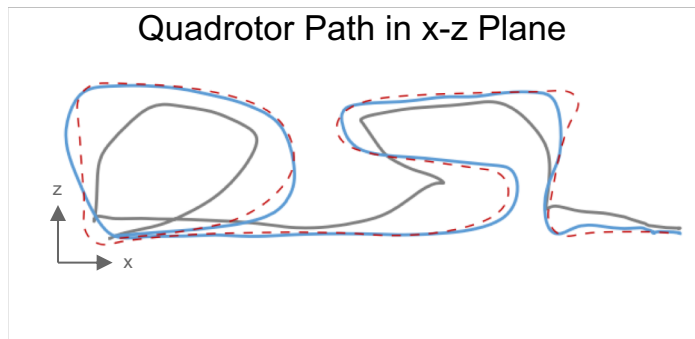


## % Error Reduction Distribution

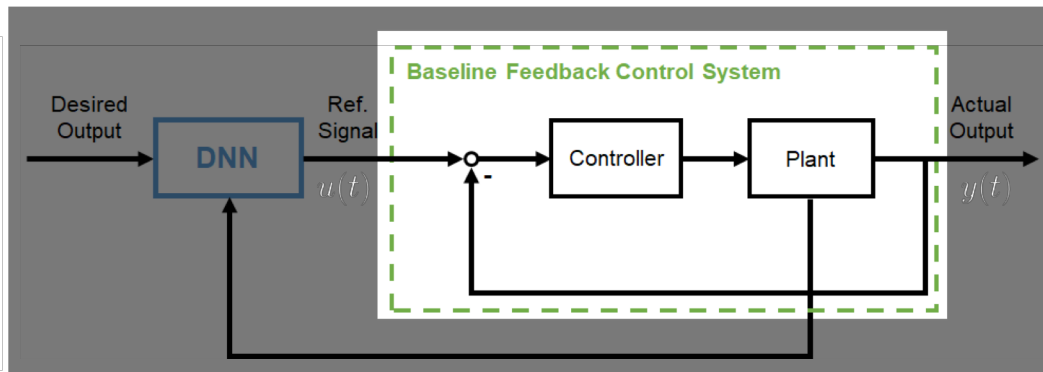


From ICRA 2017

# Control theory guides us towards more efficient training



- - Desired
- DNN (Trial-and-Error)
- Baseline



## Baseline System Dynamics

Linear

$$\begin{aligned} x(t+1) &= Ax(t) + bu(t) \\ y(t) &= cx(t) \end{aligned}$$

Nonlinear

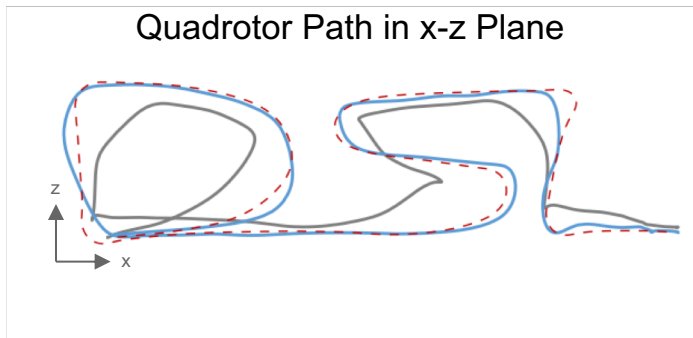
$$\begin{aligned} x(t+1) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{aligned}$$

## Platform-Independent Formulation

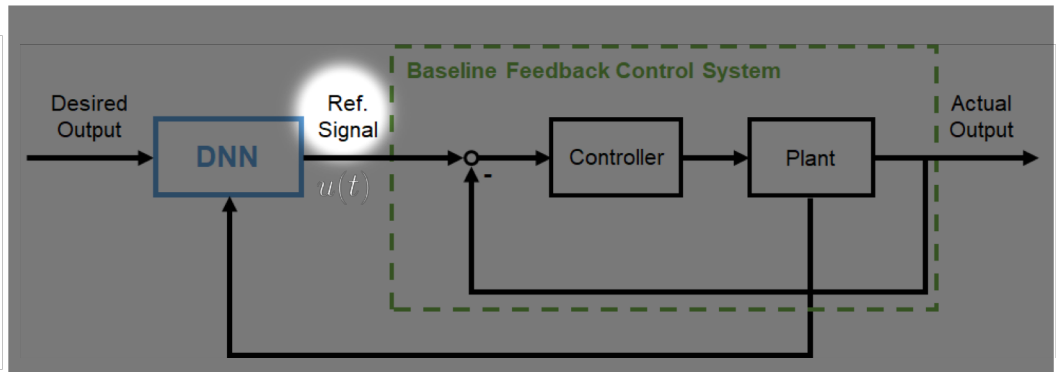
S. Zhou, M. K. Helwa, and A. P. Schoellig

*"Design of Deep Neural Networks as Add-on Blocks for Improving Impromptu Trajectory Tracking"* (CDC 2017)

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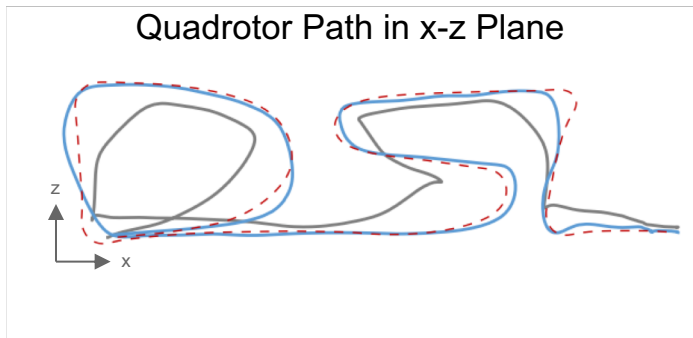
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## Ideal Control Law

Output Equation of the System's Inverse Dynamics

$$u(t) = \frac{1}{cA^{r-1}b} (-cA^r x(t) + y_d(t+r)) \quad u(t) = F(x(t), y_d(t+r))$$

# Control theory guides us towards more efficient training



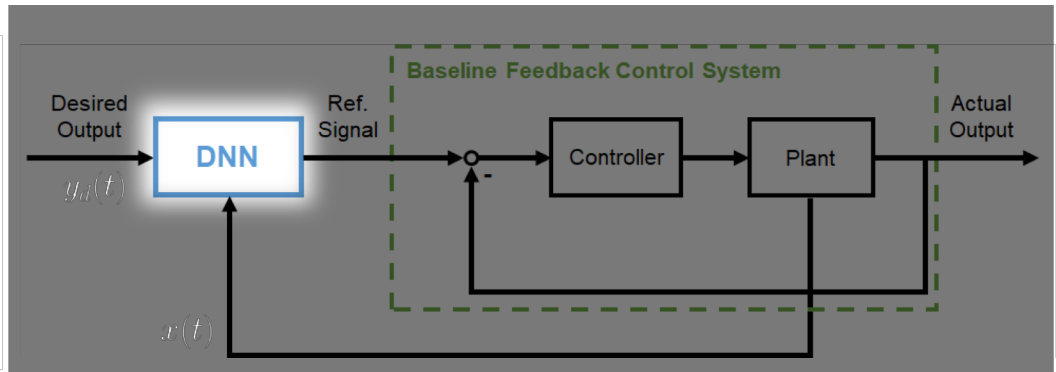
- - Desired    — DNN (Trial-and-Error)  
— Baseline

Necessary Inputs

$\{x(t), y_d(t+r)\}$

## Platform-Independent Formulation

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## Baseline System Dynamics

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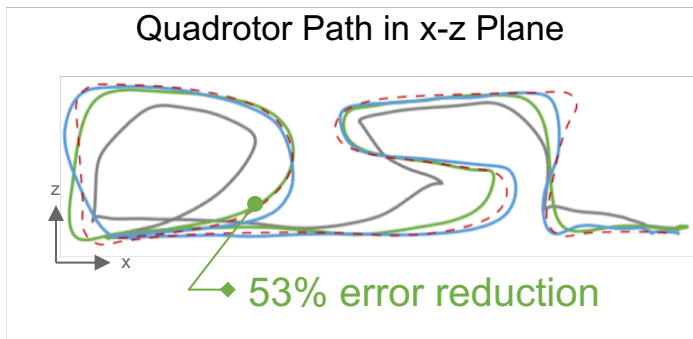
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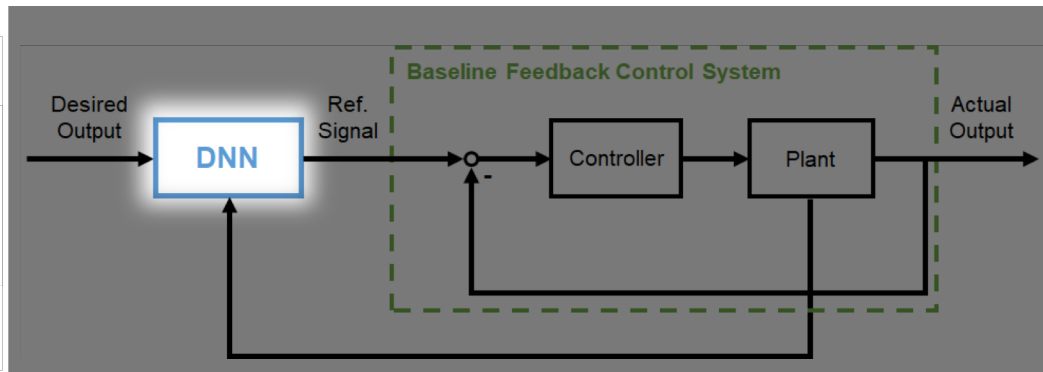
- - Desired
- DNN (Trial-and-Error)
- Baseline
- DNN (Theoretical Insights)

## Necessary Inputs

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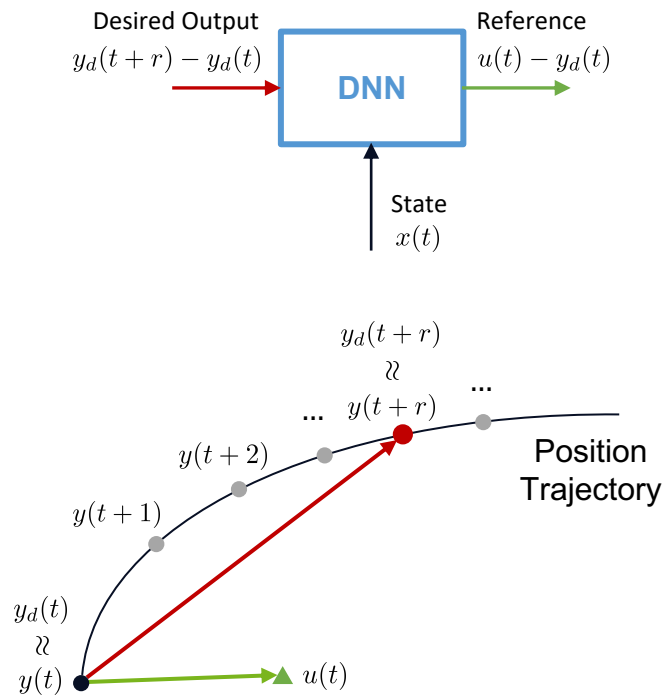


## Relative Degree $\tau$

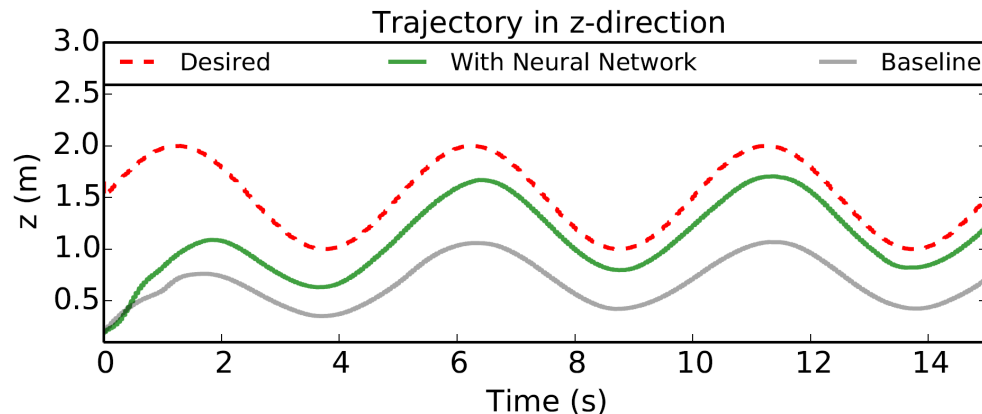
- **Inherent delay** of the baseline system, or the number of time steps between applying reference input and first seeing effects in output
- Can be experimentally identified through simple **step responses**

Similar performance (53% tracking error reduction) with **DNN input dimension reduced by 2/3**

# Condition for more data-efficient training

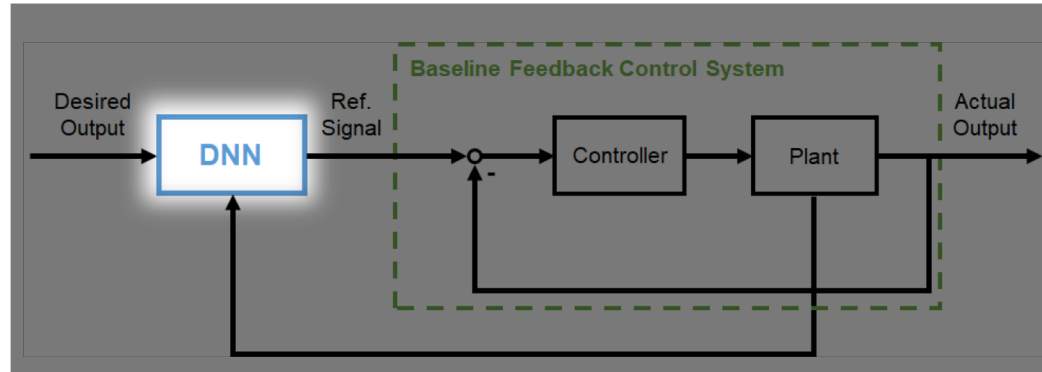


- **Difference learning scheme:** In previous work, for the quadrotor tracking problem, **relative positions w.r.t. the desired trajectory** are used to simplify the DNN training.
- **Condition:** the baseline black-box system **achieves zero steady state error for step inputs**.
- If not achieved, the underlying function becomes **one-to-many**, which cannot be learned by the DNN.





# Summary of insights



## Insight 1:

- In order to achieve unity mapping from the desired to the actual output, the DNN module can be formularized as **the output equation of the baseline system's inverse dynamics**.
- Due to the association with the inverse dynamics, the efficacy of the proposed approach relies on two necessary conditions (1) the system has a **well-defined relative degree** and (2) the system has **stable zero dynamics**.

# Summary of insights

**Insight 2:** In order to achieve unity mapping from desired output to actual output,

- a. based on the **state-space formulation**, the input features should be selected as

$$\{x(t), y_d(t + r)\}$$

can be determined from simple  
step-response experiments

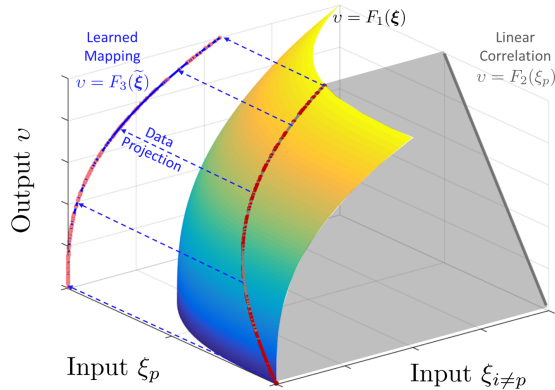
- b. based on the **transfer-function formulation** (for linear systems), the input features can be alternatively selected as

$$\{y_d(t - n + r), \dots, y_d(t + r - 1), y_d(t + r), u(t - n + r), \dots, u(t - 2), u(t - 1)\}$$

independent of state

**Insight 3:** The applicability of the data-efficient difference learning scheme relies on the condition that the baseline system achieves **zero steady state error for step inputs**.

# Direct application to non-minimum phase systems is not safe

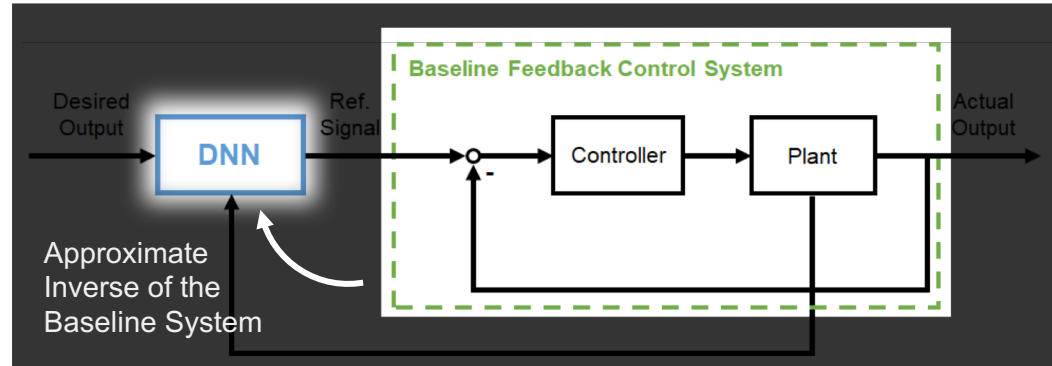


## Adaptation to Non-Minimum Phase Systems

S. Zhou, M. K. Helwa, and A. P. Schoellig

*"An Inversion-Based Learning Approach for Improving Impromptu Trajectory Tracking of Robots with Non-Minimum Phase Dynamics"*

(Submitted to RA-L and ICRA 2018)

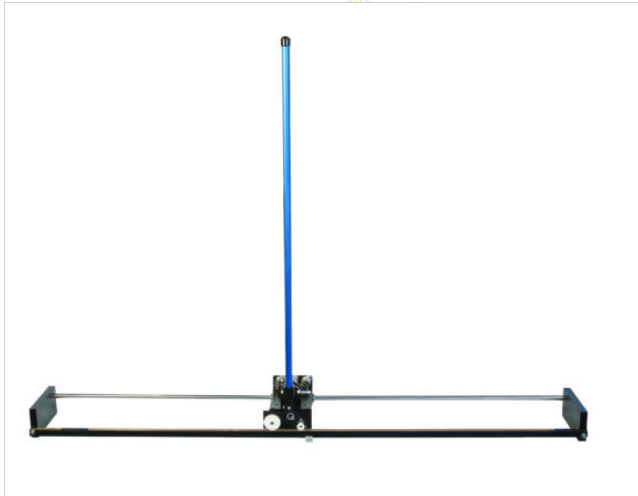


- Straightforward application does not work for **non-minimum phase systems** (i.e., systems with unstable inverse dynamics)
- Learning **stable inverse approximations** through **removing inputs** from the DNN module
- Compromise **exactness** for **stability**

# Direct application to non-minimum phase systems is not safe

## Inverted Pendulum Experiment

(Image from Quanser)

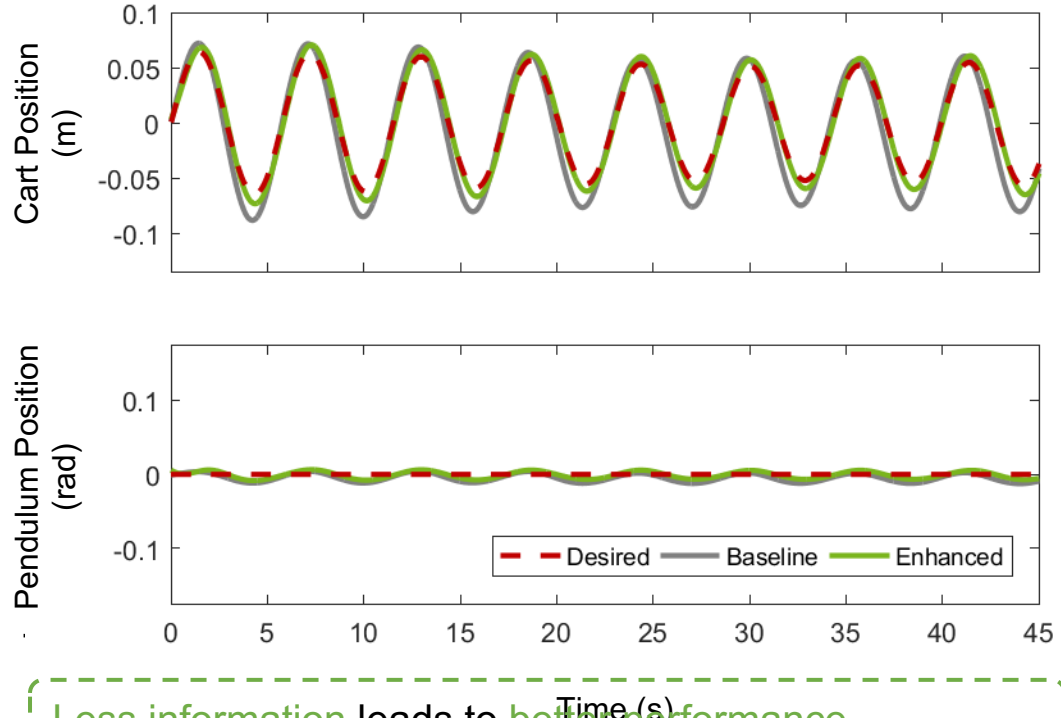


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Less information leads to better performance

# Summary of Contributions

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## First Practical Implementation

Q. Li, J. Qian, Z. Zhu, X. Bao, M. K. Helwa, and A. P. Schoellig. ICRA 2017

- Proposed **DNN as add-on block approach** for enhancing black-box tracking control systems
- Successfully tested on quadrotor vehicles for **tracking arbitrary hand-drawn trajectories**

## Current Work

S. Zhou, M. K. Helwa, and A. P. Schoellig. CDC 2017

- Provided **platform-independent formulation** of the proposed DNN-enhanced control architecture
- Proposed **efficient input selection** of the DNN add-on module for enhancing black-box systems
- Identified **necessary conditions** for the proposed approach to be effective

## Follow-up Work

S. Zhou, M. K. Helwa, and A. P. Schoellig. Submitted to RA-L and ICRA 2018

- Proposed an **approximate inverse learning** approach to extend the DNN-enhanced architecture to non-minimum phase systems



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Neural networks are **effective** for improving tracking performance of black-box control systems; control insights are important for **safe and efficient** network design.



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**Thank you!**

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